



# How should we handle missing data?

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## What is Missing Data?

- Item missing
- Unit missing



## What is Missing Data? (Theory)

- MCAR
- MAR
- MNAR



## Missing Completely At Random (MCAR)

- Suppose that only one variable  $Y$  has missing data, and that another set of variables represented by the vector  $X$ , is always observed (Marsden and Wright, 2010). The data is MCAR if the probability that  $Y$  is missing does not depend on either  $X$  or  $Y$  itself.
- Example: An example of MCAR is a weighing scale that ran out of batteries. Some of the data will be missing simply because of bad luck. (Van Buuren & Van Buuren 2012)



## Missing At Random (MAR)

- Data on  $Y$  is considered MAR if the probability that  $Y$  is missing does not depend on  $Y$ , once we control for  $X$ . MAR allows for missingness on  $Y$  to depend on other variables so long as it does not depend on  $Y$  itself.
- Example: Women are less likely to report their incomes – regardless of what their income actually is.



## Missing Not At Random (MNAR)

- Means missingness depends on unobserved values (Silverwood et al. 2021), and that the probability that  $Y$  is missing depends on  $Y$  itself, after adjusting for  $X$  (Marsden and Wright, 2010). For example, people who have been arrested may be less likely to report their arrest status.
- Example: People with low incomes do not answer the income question.



## Why Should we care about Missing Data?

- ‘Flipping’ – where missingness flips the substantive significance of a finding from positive to negative or vice versa
- ‘Flopping’ – where missingness minimises or over-emphasises the size of the substantive finding
- ‘Flip-Flopping’ – where missingness flips the substantive significance and minimises/over emphasises the result



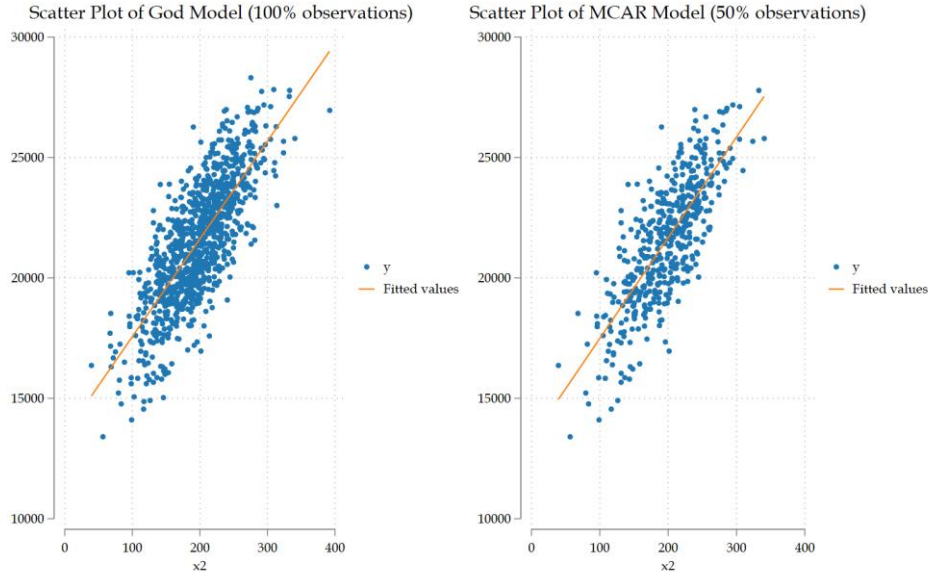
## A quick working example

- A simple bivariate regression model is simulated
- The first model has a continuous dependent variable and a continuous independent variable with  $n=1000$
- Model is injected with both a MCAR and a MAR mechanism to demonstrate potential issues



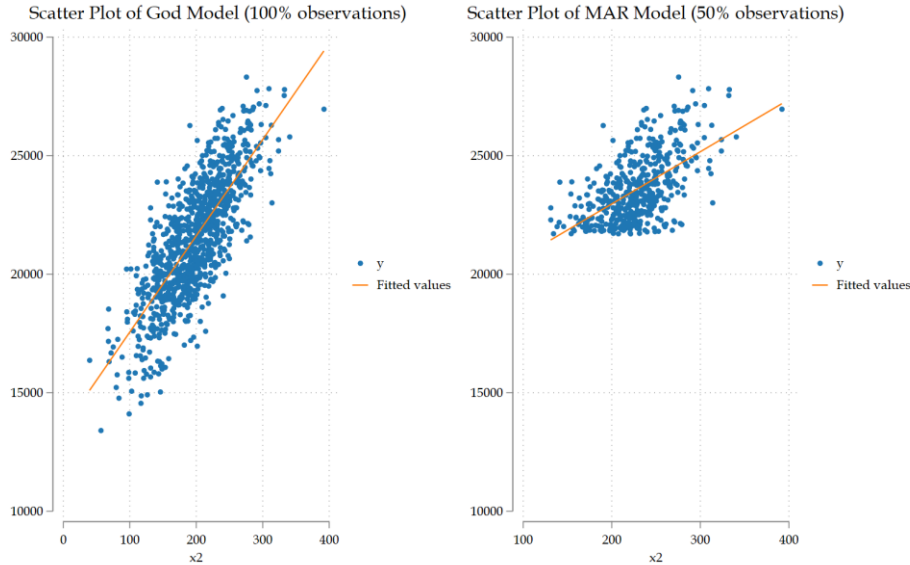


## 'God' Model versus MCAR





## 'God' Model versus MCAR





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What does this all mean?

- We can't ignore missing data
- And yet most studies do
- “I've looked at the missingness in my data and confirmed there will be no bias...”

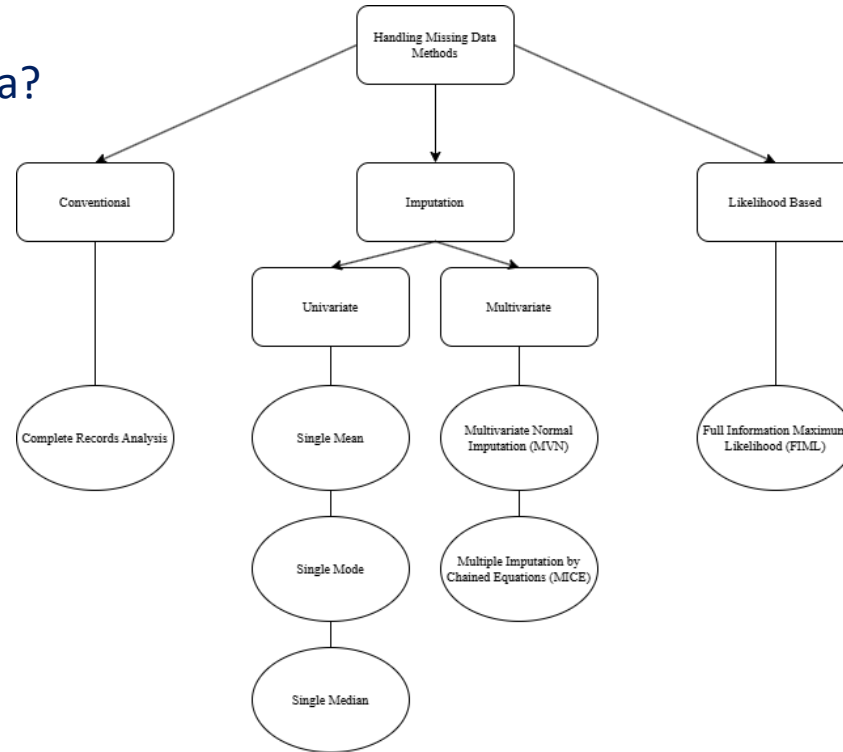


## How to handle missing data?

- Several approaches
- Some good
- Some bad
- Some ugly



## How to handle missing data?





## The Bad

- Listwise Deletion
- This just ignores the issue



## The Ugly

- Recoding Missingness to a single value
- Say you have a binary independent variable where all missingness occurs in model
  - Code all missingness = 0 in that variable
  - Code all missingness = 1 in that variable



## The Ugly

- Single mean/modal/median imputation





## The Ugly

- Multiple Imputation with zero auxiliary variables



## The Good

- Full Information Maximum Likelihood (FIML)
  - (Or MLMV in stata)
- Uses SEM framework
- Can't use for non-linear models in Stata (Can in MPLUS)



## The Good

- Multiple Imputation with auxiliary variables



## Multiple good ways to handle missing data?

- Multiple Imputation versus FIML



## Simulation Study

- N=1000
- 1 continuous dependent variable + 3 independent variables
- Missingness introduced into x2 variable
- Different handling missing data methods then assessed



## Variables

- $X_1 = n(1000)$  means(40) sds(12)
- $X_2 = n(1000)$  means(200) sds(50)
- $X_3 = n(1000)$  means(150) sds(5)
- $y = 30 * x_1 + 40 * x_2 + 50 * x_3 + \text{rnormal}(5000, 1500)$



## Missing Mechanisms

- MCAR =
- `gen rmcar = rbinomial(1, 0.5) // MCAR: 50% chance of missingness (binary random)`
- `replace x2 = . if rmcar == 0 // Set x to missing where rmcar == 0`
  
- MAR =
- `gen prob_mar = logistic(y-21791)`
- `gen rmar = 0 if prob_mar==0`
- `replace x2 = . if rmar == 0 // Set x to missing where rmar == 0`



## Models

- 1) God Model
- 2) SEM God Model
- 3) MCAR Model
- 4) MAR Model
- 5) Single Mean Imputation Model
- 6) FIML Model
- 7) 10 imputation no auxiliary Model
- 8) 10 imputation auxiliary Model
- 9) 100 imputation auxiliary Model





**Table 1: Simulation Regression Models Using a MAR Principle**

	Complete Records 'God Model'		Complete SEM		MCAR Introduced		MAR introduced		Single Use Mean Imputation		FIML		Imputed with no auxiliary variables and 10 imputations		Imputed with 10 imputations		Imputed with 100 imputations	
	<i>Coef.</i>	<i>95% CIs</i>	<i>Coef.</i>	<i>95% CIs</i>	<i>Coef.</i>	<i>95% CIs</i>	<i>Coef.</i>	<i>95% CIs</i>	<i>Coef.</i>	<i>95% CIs</i>	<i>Coef.</i>	<i>95% CIs</i>	<i>Coef.</i>	<i>95% CIs</i>	<i>Coef.</i>	<i>95% CIs</i>	<i>Coef.</i>	<i>95% CIs</i>
<b>Independent Variable 1</b>	30.01	[22.25, 37.77]	30.01	[22.27, 37.75]	29.96	[18.95, 40.99]	18.44	[9.67, 27.20]	33.76	[21.31, 46.20]	29.92	[19.84, 40.00]	29.55	[18.62, 40.47]	31.36	[21.51, 41.21]	24.96	[15.55, 34.37]
	(3.96)		(3.95)		(5.62)		(4.47)		(6.35)		(5.14)		(5.57)		(5.03)		(4.80)	
<b>Independent Variable 2</b>	40.02	[38.15, 41.88]	40.02	[38.16, 41.88]	40.03	[37.39, 42.67]	24.76	[22.06, 27.45]	25.40	[19.94, 30.86]	40.03	[37.51, 42.54]	41.44	[38.89, 43.99]	41.51	[38.88, 44.13]	38.61	[36.20, 41.02]
	(0.95)		(0.95)		(1.35)		(1.38)		(2.78)		(1.28)		(1.30)		(1.34)		(1.23)	
<b>Independent Variable 3</b>	49.88	[31.23, 68.53]	49.88	[31.27, 68.49]	51.30	[24.88, 77.71]	30.23	[9.29, 51.18]	56.30	[26.41, 86.19]	49.55	[25.48, 73.61]	71.26	[50.46, 92.06]	44.77	[20.85, 68.69]	38.68	[16.16, 61.12]
	(9.52)		(9.50)		(13.48)		(10.69)		(15.25)		(12.28)		(10.61)		(12.21)		(11.49)	
<b>Number of observations</b>	1000		1000		499		489		1000		1000		1000		1000		1000	
<b>R<sup>2</sup></b>	0.68				0.66		0.43		0.12									

Data Source: Simulation using a MAR principle. 50 per cent missingness introduced.



## What does this all mean?

- 1) God Model – perfect ideal model
- 2) SEM God Model – same as above
- 3) MCAR Model – inflated standard errors
- 4) MAR Model – big substantive issues
- 5) Single Mean Imputation Model – x2 issues, massive 95% CIs
- 6) FIML Model – Great!
- 7) 10 imputation no auxiliary Model – inflated x3 values
- 8) 10 imputation auxiliary Model – Great!
- 9) 100 imputation auxiliary Model – Great!



## Conclusion

- No missing data is always the dream
- Dream is never reality
- Have to check for missing mechanisms
- If MCAR -> carry on, if MAR or MNAR -> look to handling missing data methods
- Using 'bad' methods is sometimes as bad as doing nothing!
- No difference in efficiency between FIML and MI approaches
- Use the method that best suits your data
  - FIML is very restricted in most software, MICE is ubiquitous and easy to implement



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Thank You

- Any Questions?



- Van Buuren, S. and Van Buuren, S., 2012. *Flexible imputation of missing data* (Vol. 10, p. b1182). Boca Raton, FL: CRC press.